

Recreational folding in the 20th century: Between Beloch and Huzita

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Abstract

It is well known that the problem of the duplication of the cube is equivalent to constructing a segment of length $\sqrt[3]{2}$ (given another segment with length 1). It is also well known that if one constructs segments only with straightedge and compass, then constructing a segment, whose length is $\sqrt[3]{2}$, is impossible. This was proven by Pierre Wantzel in 1837, and almost immediately became known in the mathematical community. But in 1934 the Italian mathematician Margherita Piazzolla Beloch proved that when one uses only paper and folds it, then one can in fact construct a segment of length $\sqrt[3]{2}$. [Beloch 34; Hull 11].

Beloch's results were completely forgotten, until their rediscovery by Humiaki Huzita during the second half of 1980s. The paper will however claim that mathematical paper folding as such did not vanish from the mathematical scene during these decades – but rather remained active within recreational mathematics. However, to understand the background for this, let us recall briefly why Beloch's discoveries were forgotten or ignored. One may suggest that during the 1930s, when the construction of segments was not in the center of the mathematical interest, Beloch was certainly making an exception. Other sociological reasons also contributed to this disappearance: Beloch's papers might have been considered as belonging solely to the domain of mathematical pedagogy, whereas the social and political atmosphere in Italy during the 30s and the 40s of the 20th century also prevented the dissemination of Beloch's discoveries.¹

The question that rises is whether indeed paper folding based geometry was completely forgotten until the 1980s, that is, until the (re)discovery of Jacques Justin [1984; 1989] and Huzita [1989] that paper folding based geometry is stronger than compass-and-straightedge based geometry.² In this paper I would like to claim the contrary and show that paper folding was considered legitimate mathematical activity within the culture of *recreational mathematics* during these “lost decades” of the 20th century. In that way, one

¹ In addition, Beloch, as a woman, was by all means belonging to a minority, and this certainly made it more difficult for her results to be well-accepted as contributing to the mathematical research. Also, Fascism deprived mathematics in Italy from the works of some of its major contributors, especially after the 1938 racial laws, and the connections with the mathematical communities outside of Italy were heavily strained and damaged. See: [Guerraggio and Natasi 2006, pp. 243-281]

² For an elaborate analysis of the results of Justin and Huzita during the 1980s, see [Friedman 2018, chapter 5].

might say, several – though not all – of the mathematical discoveries and constructions enabled by paper folding were preserved.

Indeed, already before the publication of Beloch's papers, recreational mathematics took paper folding seriously as mathematical activity. This is to be seen with the 1901 book of Wilhelm Ahrens (1872–1927) *Mathematische Unterhaltungen und Spiele* [Ahrens 1901], which was reprinted and edited in several subsequent editions. A revised edition of the book was the successful book *Mathematische Spiele*, which had five editions between 1907 and 1927. Ahrens was clearly influenced from Tandalam Sundara Row's book *Geometric Exercises in Paper Folding* [Row 1893], as Ahrens himself clearly refers to Row and presents several of the exercises that appear in Row's book, but now under the context of recreational mathematics.

Several recreational mathematics books followed or were inspired by Ahrens's presentation of Row's exercises, for example, regarding the construction of regular polygons with folding. To give an example: the book *Matematica dilettevole e curiosa* (Delightful and Curious Mathematics), which came out in Italy in 1913, written by Italo Ghersi, was also influenced from Ahrens [Gheresi 1913]. More important to the dissemination of folding based mathematics however is the collection of mathematical puzzles and riddles by Henry Dudeney (1857–1930). His 1917 book *Amusements in Mathematics* already contained several folding exercises, while his 1931 book contains several other exercises [Dudeney 17; 31]. Essential to the spread of paper folding as a mathematical activity is the publication in 1967 of the book *536 Puzzles & Curious Problems*, edited by Martin Gardiner, a book which concentrated hundreds of Dudeney's exercises [Dudeney 67]. The folding exercises that appear in the books show the various influences and traditions to be found in Dudeney's work: the famous problem of knotting-folding the regular pentagon (see: [Sharp 2016]), as well as Ahrens' and Row's work, and of course Dudeney's own mathematical discoveries. Indeed, the collection presents new problems, such as determining by folding the largest equilateral triangle that could fit in a given square. Needless to say, Martin Gardner's own work also contributed since the 1950s to the mathematics of folding, especially regarding flexagons [Lister 2005]. The paper aims therefore to show that several mathematical traditions of mathematical paper folding, which existed during the late 19th century and the beginning of the 20th century, were preserved with the works of Ahrens, Dudeney and Gardiner.

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